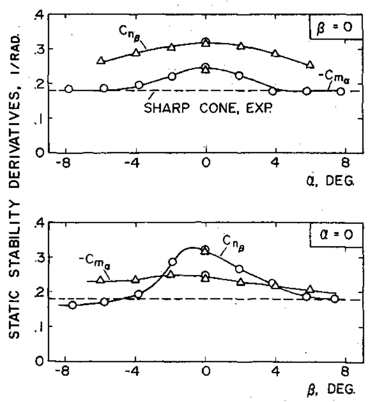

Fig. 3 Static stability derivatives of Model A, (moment coefficients are referenced to base area and base diameter, moment center at $l/L = 0.65$).

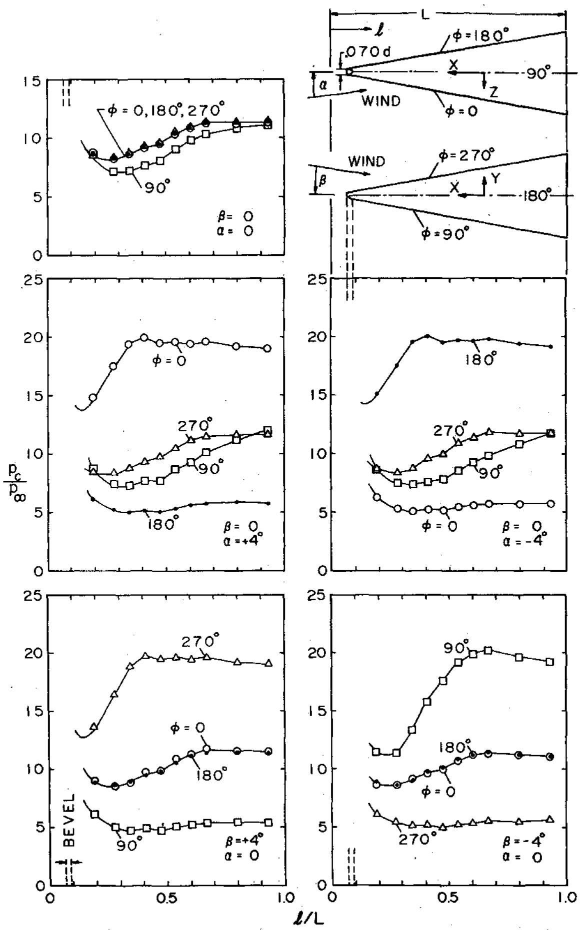

Fig. 4 Model B and pressure distributions at various angles of attack and sideslip.

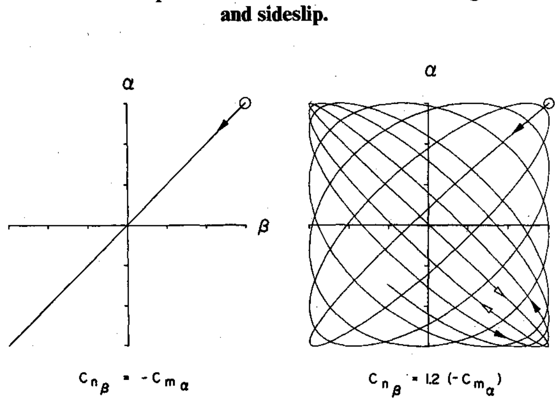

Fig. 5 Effect of equal and nonequal derivatives in pitch and yaw on the motion in the $\alpha - \beta$ plane, ($I_y = I_z$, $p = 0$, $\alpha_0 = \beta_0$, $\dot{\alpha}_0 = \dot{\beta}_0 = 0$).

Equation (2), however, does not apply for the $-y$ side, where the shock radius is again governed by the planar slanted nose cut. An asymmetric surface pressure distribution results from the rotational asymmetry of the shock. For $\alpha = \beta = 0$ the largest overexpansion is observed along the ray $\phi = 90°$, which is downstream of the low drag portion of the nose. The centroid of the resulting side load distribution may coincide with the center of gravity, in which case the vehicle would not trim. If mass asymmetry is also present, the side force combined with the c.g. offset along the z axis will provide a roll torque that would spin a rolling vehicle up or down. This example shows, that the unfavorable condition of "roll through zero" can occur without trim. The rotational symmetry of the stability derivatives is again destroyed by the small configurational asymmetry of the nose; $C_{n_\beta} = 1.17(-C_{m_\alpha})$ was measured for $\alpha = \beta = 0$.

The effect of unequal values of C_m and C_{n_β} on the angular motion is illustrated in Fig. 5 by the very simple case of a nonrolling vehicle with $I_y = I_z$. Damping is neglected and zero trim is assumed. The initial conditions are $\alpha_0 = \beta_0$, $\dot{\alpha}_0 = \dot{\beta}_0 = 0$. When $C_{n_\beta} = -C_{m_\alpha}$, constant over a small range of angles, the motion pattern in the $\alpha - \beta$ plane is a straight line, as predicted by the tricyclic theory,[4] and this pattern is repeated with every cycle of oscillation. When $C_{n_\beta} \neq -C_{m_\alpha}$, the motion pattern is a Lissajous figure that is not predictable by the tricyclic theory because its basic assumption of aerodynamic rotational symmetry does not hold. For $C_{n_\beta} = 1.2(-C_{m_\alpha})$, the period of a yaw cycle is 10% shorter than the period of a pitch cycle, and the motion character changes continuously. Sharp loops are followed by rounder loops and round ones become sharp again. Upon completion of this series of loop changes, the angular motion reverses its direction.

Physically, the investigated models are properly described as bodies of revolution having a small configuration asymmetry. Aerodynamically, however, they are not bodies of revolution because the configuration asymmetry is at the nose.

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Instability of Hypersonic Viscous Shock Layer with Finite Rate Chemistry

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Nomenclature

Re = shock Reynolds number per foot of body radius = $\rho_\infty U_\infty / \mu_1$
ρ_∞ = density of the gas in the freestream
U_∞ = flight speed
μ_1 = viscosity of the gas immediately behind the shock front
T = temperature

Received March 14, 1972. The author wishes to express gratitude to S. Wang and J. A. Fox of the University of Mississippi for many helpful discussions. This work was supported by the National Science Foundation through a grant (NSF-GK-4172).

Index categories: Boundary-Layer Stability and Transition; Reactive Flows.

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DEVELOPMENT of the interplanetary missions and the extensive use of intercontinental ballistic missiles with hypersonic flight speeds, have created a growing need for detailed studies of the physics of entry into the Earth and planetary atmospheres. Blunt bodies are widely used on noses of re-entry vehicles to alleviate heating problem. The flow in the stagnation region of a blunt body has been classified into a number of regimes by Probstein.[1] In the past decade many approximate solutions were obtained for the blunt body flowfield using various approximations. For instance, Probstein[2] used a Newtonian approximation, Levinsky, and Hoshihara[3] also treated the flow as rarified. Lighthill[4] obtained an analytical solution for the inviscid and ideally dissociating gas model. Chung[5] and Stoddard[6] obtained solutions for the viscous layer regime treating the air as a binary mixture of molecules and atoms. Blottner[7] considered a complete air model. The results of Blottner[7] are in agreement with those of Chung and Stoddard. Experimental investigations were also reported, for instance the work reported by Kondo, Tamaki, and Kawamura[8] involved experimental investigation of blunt body flowfield. The experimental results are in a reasonable agreement with the theoretical results. However, considerable uncertainty exists in the experimental results also due to the instrumentation errors and boundary-layer growth in the wind tunnel, etc.

However, the solutions above are only idealized solutions; in practice there may be disturbances in the flowfield such as radiation, injection of foreign gas, surface roughness of the body, and chemical effects due to the ablating species, etc. Sometimes incident high-frequency pulses may cause instability in the flowfield. In fact there may be many sources of disturbances. The disturbances may cause instability in the flowfield. The instability need not necessarily lead to turbalence, but may lead to another state of laminar motion. Wang[9] studied the stability of re-entry flowfield using perfect and inviscid gas model, and concluded that Taylor-Gortler type of disturvances may cause the instability of the flowfield. Dixit [10] studied the hydrodynamic stability of the flowfield near the stagnation stream line of a blunt body, using the inviscid and ideally dissociating gas model. Dixit found that the flowfield is unstable for Taylor-Gortler type of disturbances also.

In the present analysis, the stability of hypersonic viscous shock layer with finite rate chemistry, near the stagnation streamline of a blunt body entering the Earth's atmosphere has been investigated with respect to infinitesimal Taylor-Gortler type of disturbances. The shock layer was found to be most likely unstable. The customary practice of treating the air as a binary mixture of molecules and atoms is followed in the investigation reported here. The viscous effects are assumed to be felt throughout the shock layer except in the immidiate viscinity of the shock wave which can still be considered as a thin discontinuity. Pressure and thermal diffusion are assumed to be negligible. The vibrational and the rotational degrees of freedom are assumed to be in equilibrium with the translational degrees of freedom everywhere in the shock layer, and the electronic degrees of freedom are considered to be unexcited. The shock wave is assumed to be concentric with the spherical stagnation point. Viscosity is assumed to be proportional to $T^{2/3}$. The specific reaction rates proposed by Chung[11] are used for the purpose of computing the net production of atomic species. The conservation equations are transformed into the blunt body coordinate system.

Taylor-Gortler type of infinitesimal disturbances in all the dependent variables, are introduced into the set of governing equations. The steady-state components automatically satisfy the steady-state equations. The set of equations are then linearized by neglecting second and higher powers in disturbance quantities. The steady-state solutions suggested by Stoddard[6] are used in the present analysis. The unknown variables in the set of equations are reduced to four by the method of substitution and elimination. Two of the resulting equations are of fourth-order and the other two are of third-order differential disturbance equations, and all of the four equations are coupled. A transformation technique suggested by Kurtz[12] is used to transform the differential system into a discrete algebric model. The problem then is an eigenvalue problem with complex polynomial matrix elements. The resulting (20 × 20) determinant of the characteristic equation has been solved for eigenvalues using iterative procedures, at the computer center of the University of Mississippi. Traub's[13] technique of iterative root finding in the complex plane, which uses a second-order Newtonian interpolation, has been used in the investigation reported here. To avoid overflow and underflow during determinant evaluation, the matrix elements are scaled, scale factor depending on the maximum of the absolute values of the elements in each row.

A numerical search failed to yield eigenvalues under neutrally stable conditions. However, eigenvalues with large amplification factors were detected suggesting that the shock layer is most likely unstable. It is of particular interest to note that the growth rate of disturbances decreases with decreasing shock Reynolds numbers. An increase in freestream Mach number causes growth rate to decrease. In general, for a given altitude the growth rate decreases for increasing flight speeds, suggesting that the viscosity and the dissociation in the shock layer have some stabilizing effect.

Since large values of amplification factor were detected, the present analysis fails to predict the stability of the flow system after a short interval of time. In order to understand the flow mechanisms more clearly, the nonlinear disturbance equations should be solved. Since the flow system is less unstable for smaller shock Reynolds numbers, the flow system might be stable in more rarified regimes such as occurs in the incipient and fully merged layer regimes. The present analysis can be extended to study the stability of the flow system in planetary atmospheres.

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